The Use of Coulomb’s Law Can Account for the Attractive Force between Electromagnets

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***Abstract*-This paper demonstrates that Coulomb’s Law can account for the attractive force between two electromagnets, provided the respective currents have the same direction of rotation, the electromagnets being aligned along the same axis. The work is based on earlier research showing that Coulomb’s law can account for the forces between collinear currents in Ampère’s bridge. This was done by recognising the need for analysing the delay of action that can be ascribed the different propagation time for the action between parts of two currents due to their relative distance. This will cause the stationary positive ions bound to the metal lattice in an electric conductor to give rise to a different strength of the force field than the moving electrons. The difference between these fields constitutes the so-called magnetic force. It is also necessary to take into account the Lorentz transformation of lengths according to the Special Relativity theory in order to predict the attractive force between two parallel currents. This result explains why two electromagnets carrying currents with the same rotational direction attract each other. This is shown for the simplistic case of one-winding coils.**

***Keywords*- AmpÈRe’S Law; Coulomb’s Law; Propagation Delay; Electromagnets; AmpÈRe ’S Bridge; Lorentz Force; Retarded Action; Special Relativity Theory; Lorentz Transformation**

I. INTRODUCTION: MOTIVES FOR ANALYSING ELECTRIC CIRCUITS AGAIN

It has been widely believed that basic electricity and magnetism was dealt with and finished until the end of the 19th century, though completed by the Special Relativity Theory at the beginning of the 20th century.

It is also widely believed that all the ‘great masters’ of electricity, such as Cavendish, Coulomb, Ampère, Maxwell, Lorentz, Einstein et al all agree on basic matters. The new successive generations have supposedly only added some new features though defending principally the old theories.

Experiments that cannot be explained using the Lorentz forces, such as Ampère’s bridge, have again awakened interest in studying the basics of electromagnetism [1-4]. However, the attractive force between two parallel currents has thus far not been satisfactorily explained to be a consequence of Coulomb’s law, but in fact, this has already been done [5, 6]. This study will describe how the results shall best be applied to electromagnets.

II. METHODOLOGY

A. *Extending the Use of Coulomb’s Law*

Since Coulomb’s Law is very well corroborated, beginning with Cavendish [7, 8] in the case of stationary charges, one may very well claim it to be a natural law. Being that, it must of course also be valid for any type of application, as in the case of moving charges and currents carried by electric conductors. The main problems in this respect are to define the geometrical relations with respect to propagation delay, basing this on the velocity of light as the propagation velocity, thereby including the effects of the Lorentz transformation of the lengths of the charge packets according to the Special Relativity theory. An extensive analysis of how propagation delay shall be treated in a mathematically strict way has been done already [1], though still without taking into account the effects of the Special Relativity theory. Since the paper is treating current carrying conducting devices, one would perhaps expect that the Special Relativity theory would not have any substantial effect on the result. This, however, is not the fact, when applied to two parallel conductors. Assuming that propagation delay is an effect that exists independently of the Lorentz transformation [9], they may be treated separately but combined finally in order to attain the correct expression for the force.

B. *Propagation Delay*

It is reasonable to assume that there will always appear a propagation delay between two charges interacting with each other and that it is the speed of light c that sets the conditions for this. The idea of ‘retarded action’ is otherwise already established within electrodynamics, expressed through the so-called Liénard-Wiechert potentials [10, 11]. However, the derivation of these potentials for the case of continuously distributed charges, as in the case of conductor currents, is inherited with a grave mathematical fault [5]. Hence, the derivation of electric and magnetic fields, which are expected to follow, if differentiating the potentials further, cannot be done. On the contrary, the correct interpretation of the geometry of the propagation delay of the Coulomb field with respect to the four combination of positive and negative charges makes it possible to derive the appearance of a force between two electric currents, carried by an electric conductor, without resorting to the Lorentz force. In addition, the theoretical foundations behind the original derivation of the Lorentz force has been criticised elsewhere [12]. This was first done without needing to apply the Special Relativity Theory, but in this paper it has been taken into account.

Using the method referred to above [1] leads us to the following expression for the force between two electric conductors carrying a respective current I_1 and I_2:

$$\frac{d^2\vec{F}}{dx_1 dx_2} = (\mu_0 \mu\ I_1 I_2 \cos\theta \cos\psi)\vec{u}_R / 4\pi R^2 \quad (1)$$

Fig. 1 is illustrating two straight conductors with arbitrary directions.

If the currents are parallel to each other, the angles that a straight line gives rise to, when crossing the currents will be equal, i.e. $\theta = \psi$, and hence,

$$\frac{d^2\vec{F}}{dx_1 dx_2} = (\mu_0 \mu\ I_1 I_2 \cos^2\theta)\vec{u}_R / 4\pi R^2 \quad (2)$$

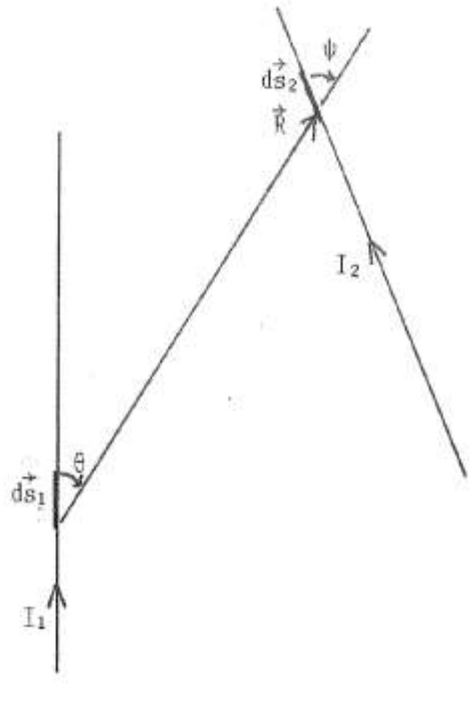

Fig. 1 Two straight conductors carrying a respective current

Basically, by using this formula it was possible to calculate the repulsive force between the two parts of Ampère’s bridge.

A model showing the principal features of a set of Ampére’s bridge is shown in Fig. 2.

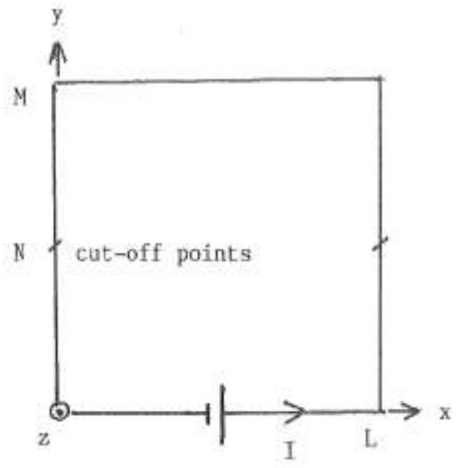

Fig. 2 Schematic figure showing a set of Ampère's bridge [1]

Using the same expression, but now evaluated for two parallel conductors, the two currents directed opposite to each other in the actual application of Ampère's bridge, the dominating term of the force becomes [6, 13].

$$\frac{d^2\vec{F}}{dx_1 dx_2} \bullet \vec{u}_R = I_1 I_2 (-2\sqrt{1+(\frac{L}{M})^2} + 4 - \frac{2M}{\sqrt{L^2+M^2}}) \tag{3}$$

If making the distance between the parallel conductors infinitesimally small, the contributions from other parts of the circuits can be neglected. This may be expressed as

$$\lim_{L\to\infty, M\to 0} \frac{d^2\vec{F}}{dx_1 dx_2} \bullet \vec{u}_R = I_1 I_2 (-2\frac{L}{M}) \tag{4}$$

A practical model, still using the basic geometry of Ampère's bridge, is shown in Fig. 3.

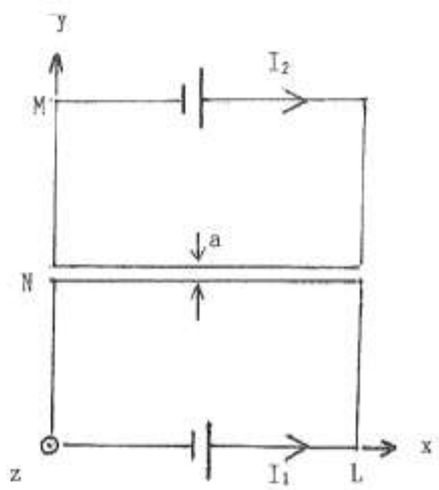

Fig. 3 Two parallel conductors using the shape of Ampère's bridge illustrating two practical closed circuits containing respective current [14]

But if the directions of the currents are the same, the sign becomes positive, since in the case analysed the directions were opposite to each other. This implies that without the application of the SRT the negative sign of the force between parallel currents cannot be predicted.

C. Lorentz Transformation

An effort has also been made to calculate the electromagnetic force between two parallel electric currents, separated by a distance that is to be regarded as short in comparison with the other dimensions of the electric circuit. This was, however, not successful; the predicted force being repulsive, contrary to experimental evidence [6, 13].

Inevitably, the Special Relativity theory must also be taken into account, provided it has the ability to affect the result. An initial effort has already been made in these papers [6, 13]. However, the analysis was not fully concurrent with respect to the Lorentz transformation of lengths. This will be corrected below. The fact that the moving charge elements containing electrons must appear to be shorter according to observers in an inertial system K in the Standard Configuration, observing that they move with the velocity v than their length indicates according to what is observed in the inertial system K', where they are regarded to be at rest, implies both a change of the charge density and of the length vector between the two conductors with respective charge element. For simplicity, here we will only demonstrate the scenario when the electron velocities of respective conductor are equal, i.e. equal v.

Four combinations of charges will be presented involving the positive and negative charges of the first conductor respective of the positive and negative charges of the second conductor. The interaction between exclusively positive or negative charges does not imply any change of charge density and distances.

The net charge within a given interval will however remain unchanged, since, if expressed as

$$\Delta Q_1 \cong \frac{dQ_1}{dx_1}\Delta x_1 \tag{5}$$

the Lorentz transformation of the length element Δx_1 being

$$\Delta x_1' = \frac{\Delta x_1}{\gamma} \tag{6}$$

and, since that incremental distance appears both in the numerator and the denominator, they cancel.

The 'Standard Configuration' is being used, with two inertial systems, K according to which the electrons of the first current are moving with velocity v_1, but for simplicity the velocities will be regarded as equal $v_1 = v_2 = v$ when the γ factor is concerned in the following. K' is the inertial system according to which the electrons are not moving.

The change that remains to be considered is the Lorentz transformation of the length vector between the two charge elements. Admittedly, the angles θ and ψ are also affected due to the Lorentz contraction of one coordinate, but since there is already a multiplicative factor $\frac{v}{c}$ immediately in front of both the $\cos\theta$ and the $\cos\psi$ terms, the $(\frac{v}{c})^2$ embedded in the γ term will have no substantial effect on the expression, as far as cases in which $v << c$ are taken into account, as is the case with conductor electrons. Regrettably, this discussion was omitted in both the cited papers. In these circumstances

$$\cos\theta = \frac{x}{R} \tag{7}$$

and

$$\cos\theta' = \frac{x'}{R'} \tag{8}$$

will be treated as equal. The same holds for

$$\cos\psi = \frac{x}{R} \tag{9}$$

and

$$\cos\psi' = \frac{x'}{R'} \tag{10}$$

If either of the charge elements interacting with each other is moving, its length must be divided by the Lorentz factor γ. This will give rise to a Lorentz modified length

$$R' = ((\frac{x}{\gamma})^2 + y^2 + z^2)^{½} \tag{11}$$

instead of the length when no movement takes place:

$$R = (x^2 + y^2 + z^2)^{½} \tag{12}$$

To conclude, if electrons moving with a velocity v are studied in both conductors, the distance vector will be R, because the two Lorentz contractions on both ends of the array connecting the two charge elements cancel each other, even though the charge elements are both Lorentz contracted.

This gives rise to the following equation:

$$(\frac{d^2\vec{F}}{dx_1 dx_2})total = \frac{\mu_0 I_1 I_2 y}{4\pi R^3}\cdot(-2\cdot\cos^2\theta) \tag{13}$$

Apparently, the force is now negative, i.e. attractive, which is in accordance with observation.

III. APPLICATIONS

A. The Attraction between Two Electromagnets Explained in the Most Simplistic Condition

Two electromagnets carrying both a respective current, constructed in the shape of two coils, aligned along the same axis are being studied. The most simplistic condition consists of coil made of one single winding, situated in the vicinity of another coil of the same shape. This is illustrated in Fig. 4. The distance d between the two coils is so small that its relation to the circumference C of one winding, whose radius is r, where

$$C = 2\pi r \tag{14}$$

can be expressed through

$$d << C. \tag{15}$$

Since in this case the ‘long distance’ is in the shape of a circle, one may raise the objection that it is not a straight line, as in the case of two parallel conductors. For practical reasons one may instead choose a quadratic shape that makes integration easier.

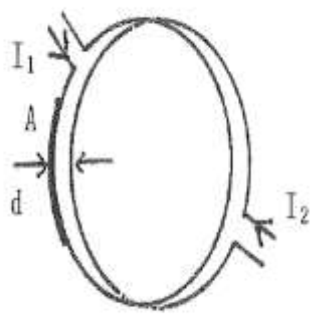

Fig. 4 Two circular one-winding coils illustrating two simplistic electromagnets in close vicinity to each other, on the same axis. Perspective figure

That means that Eq. (13) above is applicable to the two windings, but the integral has to be performed along a circle. If one prefers to work with Cartesian variables, of course the principal result may also be illustrated by two quadratic circuits according to the same conditions as above, a choice that will be demonstrated here. The length of one side of the square S will be used instead of a subsection of the winding.

Using Eq. (13) implies that for every part of the winding, there is an attractive force directed along the axis along which the coils are situated. The opposite part of a winding contributes very little to a repulsive force, but in that case the distance is much larger than when the windings are situated close to each other.

Since it is simpler to perform integrals involving functions that are linear in the Cartesian coordinates, it is more favourable to define a quadratic circuit, without loss in significance of the result.

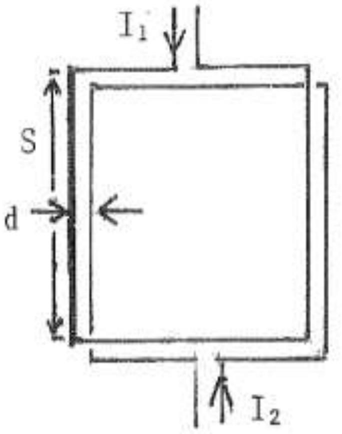

Fig. 5 Two quadratic one-winding coils illustrating two simplistic electromagnets in each other's close vicinity, on the same axis. Perspective figure

Apparently, the force between the two windings is attractive. The equation that should be used is Eq. (20) above, and, apparently, this implies a clear inverse square dependence of the distance that has to be integrated. However, the detailed calculations are up to the reader to perform, depending on which specific coils have been chosen for investigation.

In order to calculate the force between the two one-winding coils, it is possible to use the results concerning calculations on rectangular conductors that have been performed in a previous paper [14]. The basic result is that, if having two parallel conductors, only the conductors that are in close vicinity of each other contribute to the total force, provided their mutual distance approaches zero in limes. The parts of the circuit that reload the current to the voltage source, thus creating a closed circuit, don't contribute under this limes condition. In this particular case with two quadratic conductors (i.e. the two simplistic coils consisting of one single winding each) in the mutual vicinity of each other, it is only necessary to take into account the mutual force that arises at the parts of the winding that are aligned with a neighbouring winding. The result from the cited paper must be multiplied by 4, since there are four sides on a quadrat. However, as appears from that paper, the Special Relativity theory has to be taken into account and that was also done in that paper, but an updated version of that calculation is applied in this paper, Eq. (16) below. Apparently, this is a negative, i.e. attractive force. Taking Eq. (13) four times makes

$$\left(\frac{d^2\vec{F}}{dx_1 dx_2}\right)total = 4.\frac{\mu_0 I_1 I_2 y}{4\pi R^3}\cdot(-2\cdot\cos^2\theta) \tag{16}$$

That is, this expression has to be integrated once, thus achieving the dominant contribution to the force between the two quadratic coils defined above in Fig. 5.

The objection that the Special Relativity theory is the prerequisite for the result according to Eq. (16) and that circular currents imply that an acceleration takes place, which would make its use impossible, can be rejected, since other papers on the Sagnac Effect have convincingly shown that the Special Relativity theory may be used for every short part of a curved line, since every infinitesimally short piece of a curve may be treated as a straight line [15].

IV. CONCLUSIONS

From the above it is evident that the attraction between two electromagnets can also be explained using Coulomb's Law. The important precondition for finding this is that the effects of propagation delay as well as the Special Relativity theory are correctly being taken into account.

It is evident that two full-scale electromagnets will attract each other according to the simplistic analysis above, if integrating over all layers. This would also imply that the attraction between traditional magnets, consisting of magnetised matter through spins, can be explained as a consequence of Coulomb's law, as explored above.

It is also interesting to remark that other scientists are focusing their attention on basic electromagnetic interaction due to electric coils, as for example Kholmetskii et al [16].

REFERENCES

[1] J. O. Jonson, "The Magnetic Force between Two Currents Explained Using Only Coulomb's Law," *Chinese Journal of Physics*, vol. 35, no. 2, pp. 139-149, April, 1997.

[2] Pappas Moyssides, "Rigorous quantitative test of Biot-Savart-Lorentz forces," *J. Appl. Phys.*, vol. 59, no. 19, 1st January 1986.

[3] J. P. Wesley, *Progress in Space-Time Physics*, Benjamin Wesley, p. 177, ISBN 3-9800942-2-7, 1987.

[4] J. P. Wesley, Spec. Sci. Tech. 10, p. 47, 1987.

[5] J. O. Jonson, "Refutation of Feynman's Derivation of the Lienard-Wiechert Potentials," Proc. of the 10th Natural Philosophy Alliance Conference in Journal of New Energy, vol. 7, no. 3, pp. 42-44, 2003.

[6] J. O. Jonson, "The Attractive Force between Two Parallel Currents Explained by Coulomb's Law," Proc. of the 8th NPA conference, vol. 8, pp. 299-303, 2011.

[7] J. D. Jackson, *Classical Electrodynamics*, New York, USA: John Wiley & Sons, p. 5, 1975, DOI: 10.1016/0029-5582(63)90551-0. 0 0.

[8] H. Cavendish, *Electrical Researches*, Ed. J. C. Maxwell, Cambridge University Press, p. 112, 1879, DOI: http://dx.doi.org/10.1017/CBO9780511696480.

[9] Ramgard, *Relativity Theory*, Teoretisk fysik, KTH, Stockholm, Sweden, pp. 35-38, 1977.

[10] J. D. Jackson, *Classical Electrodynamics*, New York, USA: John Wiley & Sons, pp. 654-658, 1975.

[11] R. P. Feynman, (mainly Electromagnetism and Matter), Lectures on Physics, Addison-Wesley, pp. 21-9-11, 1989.

[12] J. O. Jonson, "Ampère's Law Proved Not to Be Compatible with Grassmann's Force Law," Engineering Electric and Electrical Engineering, *Electromagnetic Radiation*, book edited by Saad Osman Bashir, ISBN 978-953-0639-5, DOI: 10.5772/37978.

[13] J. O. Jonson, "A Comparison between Ampere's Law, Coulomb's Law and the Lorentz Force", Proc. of the Natural Philosophy Alliance, vol. 7, no. 8, pp. 232-239, 2010.

[14] J. O. Jonson, "The Attractive Force between Two Parallel Currents Explained by Coulomb's Law," Proceedings of the NPA, vol. 8, pp. 299-303, Eq. (7), 2011.

[15] J. O. Jonson, "The Sagnac Effect Explained Using the Special Relativity Theory," 16th Natural Philosophy Alliance Conference, Storrs, CT, United States, 2009.

[16] L Kholmetskii, O. V. Missevitch, R. Smirnov-Rueda, R. Ivanov, and A. E. Chubykalo, "Experimental test on the applicability of the standard retardation condition to bound magnetic fields," Journal of Applied Physics, vol. 101, 023532-1-11, 2007, [doi:10.1063/1.2409771].