Conventional Asynchronous Wind Turbine Models
Mathematical Expressions for the Load Flow Analysis

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***Abstract*- Asynchronous wind turbines (WT) have been among the most used type of converters in wind energy plants. Conventional asynchronous WTs were installed during the first years of wind energy research, but doubly-fed induction generator (DFIG) WTs and other types have also been added for current and future use. So, the massive presence of such machines in electrical networks means it is important to develop dynamic and steady-state models to describe their behaviour. This paper presents a review of steady-state models of asynchronous WTs for the load flow analysis (LF) that have been presented in recent years. A large number of conventional asynchronous WTs can still be found in electrical systems in many different countries all over the world. This fact constitutes a reason for the authors not to overlook them when studying the operation of such systems. In addition, there has been some discussions about these models over the last few years.**

Keywords-Asynchronous Wind Turbine; Load Flow Analysis; Newton-Raphson.

I.INTRODUCTION

The goal of this paper is to present, in the form of a brief review, some of the steady-state models for WTs the authors have been using over recent years in their work. It is a well-known fact, often described in the literature, that the presence of wind energy has been increasing over recent years. Together with this growth, detailed models have been proposed for this kind of generator.

The whole focus of this paper has been put on asynchronous WTs, more specifically on conventional ones, and on steady-state models, i.e., models for LF calculation.

Many researchers may wonder about the usefulness of developing models for such machines, when this kind of WTs will not be very common in the electrical power networks in future years. It is sure that in the future DFIG WTs, synchronous WTs or even permanent magnet synchronous generator (PMSG) WTs will be more widely used. However, the fact should not be forgotten that conventional asynchronous WTs were the most commonly used at the beginning of wind energy developments, and this fact means there are many of these machines in the electrical power networks nowadays and they will continue operating for some years. This is why the consideration of the authors is that the use of their models should not be abandoned in the calculation processes.

A well-known single phase electrical equivalent representation of a conventional asynchronous machine is given in Fig. 1.

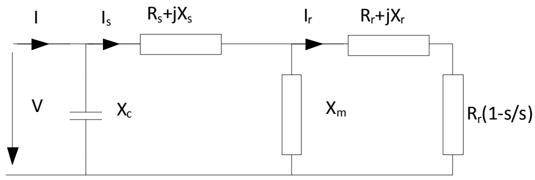

Fig. 1 Complete steady-state representation of a compensated conventional asynchronous machine

One of the possible problems when trying to simulate one of these machines following the model in Fig. 1 is that the parameters of the machine are not always known. In [1] Haque presented a proposal for estimating parameter values of the exact equivalent circuit of a three-phase induction motor, which can be a helpful complement to the simulation methods reviewed in this paper.

Together with the models that will be explained in the rest of the paper, a mention should be made of other models of interest that can be found in other papers.

Divya and Nagendra Rao [2] presented steady state models of various types of WT generating units (WTGU) with the goal of using them in deterministic and probabilistic analysis. Fixed speed, semi variable speed and variable speed WTGUs are analysed, and they propose several algorithms for the different cases.

Cutsem and Vournas [3] proposed a model where the power is calculated as a function of the slip and the voltage, assuming this slip is a known value.

Eminoglu [4] and Eminoglu *et al.* [5] proposed two different models for WTGUs based on the use of the biquadratic equation, which facilitate the computation of active and reactive power outputs for a given mechanical power and terminal voltage, avoiding computation of parameters such as slip s. The one called Model II for WTGS given in [4] can also be found in [6].

Due to the non-linear nature of the proposed models, the use of iterative processes for calculation is generally needed. In this paper an explanation is given about the models presented in [7], where the authors opted to follow two different processes, i.e., alternating the solution of, firstly, an iterative process consisting of injecting specified values of active and reactive powers, P^s and Q^s, with an LF result, and, secondly, an iterative process that solves the internal variables of the machine when a value of the terminal voltage, V, is given. In general, by coming and going between both processes, a solution is not difficult to find whenever there is one. Fuerte-Esquivel *et al.* submitted an interesting discussion to [7], which can be read in [8], where they proposed integrating both processes in one.

In a recent and more in-deph paper [9], Castro *et al.* collected similar ideas presenting mathematical models of several types of wind generators for LF analysis in networks with a large number of WTs. In that paper one of the conclusions is that the sequential method is not so effective in terms of number of iterations.

II.MODELS FOR ASYNCHRONOUS WIND TURBINES

As is perfectly known, a PQ model is a very common model for LF analysis, and many of the loads in electrical systems are systematically simulated in steady-state by using it. The assumption when using a PQ model is that active and reactive powers, P and Q, are known, so in the LF analysis they are assumed to be injected into the corresponding bus. Then, after the calculation process, which is generally iterative, the complex voltage of the bus, $\underline{V} = V\angle\alpha = Ve^{j\alpha}$, is obtained when the process converges. Here V represents the voltage root mean square (rms) value of the bus, and α represents its angle with respect to a reference, generally fixed in the slack bus of the system. We can define the complex value of power as $\underline{S} = P + jQ = Se^{j\varphi} = S(\cos\varphi + j\sin\varphi)$, where $\cos\varphi$ is a good approximation of the power factor, and the coincidence is absolute in systems where only one frequency exists, that is to say, in networks where no considerations about either harmonics or inter-harmonics have to be taken into account.

As when using PQ models, the values of the active and reactive power are input data, and a distinction is established between the so-called specified active and reactive powers, P^s and Q^s, which are input data, and the calculated active and reactive powers, P^c and Q^c, which are obtained during the iterative process. These last values are being calculated at each iteration as a function of voltages and impedances of the network, and the process is assumed to have converged when the differences denoted as $\Delta P = |P^s - P^c|$ and $\Delta Q = |Q^s - Q^c|$ are so small as desired. To be more precise, the accepted errors are generally given for the whole system, so the iterative process is finished when the errors ϵ_P and ϵ_Q are smaller than a given value, where $\epsilon_P = \sqrt{\sum_{k=1}^{n}\left(P_k^s - P_k^c\right)^2}$ and $\epsilon_Q = \sqrt{\sum_{k=1}^{m}\left(Q_k^s - Q_k^c\right)^2}$, and n and m are the number of buses in which P and Q are being calculated. It is not necessary that $n = m$ because not all the buses of a network are simulated as PQ buses. For the buses that are modelled as PV ones, there is only an equation for P, but there is not such an equation for Q.

A reflection can be made on what is the cause for thinking about a particular model for WTs instead of using the conventional PQ bus by assigning a value for P and another value for Q. And the answer can be that in the case of machines like these, some better approaches can be achieved to the conventional PQ bus model, by making use of the widely accepted steady-state model of the asynchronous generator. A new question could be why not use this advantage for more precise modelling. The rest of the paper is a discussion about possible responses to these questions.

Different approaches will be explained for steady-state modelling of asynchronous WTs, all of them having in common a PQ representation in the LF process.

In section III a first explanation about some initial possibilities with the conventional model is given. In section IV some comments about what various authors have called PX model are made. In section V models where the reactive power is a dependent value are explained. In section VI some simplifications about models presented in section V are given. In section VII a model based on an impedance transformation is presented. In section VIII a different approach to the model presented in section VII is given, by means of the theorem of the conservation of complex power. In section IX the Z model is explained. And finally in section X a summarized comparison of the models presented is given.

III.THE CONVENTIONAL MODEL

The first possible way to simulate a WT as a PQ model is trivial. It consists of assuming given values of the active and the reactive powers. In this case it is also obvious that there is not a specific model for the WT. No additional comments should be made for it as the model is widely employed for a lot of different kinds of loads in electrical power systems.

There is the possibility of expressing the active power, P, as a function of the wind speed, v_w, which can be seen in Equation 1 [10].

$$P_m = \frac{1}{2}\rho A v_w^3 c_p\left(\lambda, \beta\right) \tag{1}$$

where P is the mechanical power, ρ the air density, A the area swept by the rotor blades, v_w the wind speed, and c_p the power coefficient value, extracted from a function that depends on the tip speed ratio, λ, and the blade angle, β. The tip speed ratio is the relationship between the linear speed at the end of the blades and the wind speed, $\lambda = \frac{\Omega R}{v_w}$, Ω being the rotational speed.

The possibility of expressing the power as a function of wind speed depends on the availability of the power curve, which can be supplied by the manufacturer of the WT.

Another immediate possibility is to consider a given value of the power factor, φ, so, in this case we can assume that $Q = Ptg\varphi$. Let us remember that conventional wind turbines consume reactive power, and also that, as mentioned before, we are assuming operation with a single frequency, where $\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2+Q^2}}$.

A typical value for the power factor of a conventional asynchronous WT can be between 0 and 0.86, depending on the active power. A value of 0 would be possible if the active power were 0, but this would not be a typical way of operating, of course, and for nominal active power a value closer to the above mentioned 0.86 is more real. In the case that the machine is compensated, then it can be normal to be operating at values close to 1.

IV.THE PX MODEL

Cidrás *et. al.* proposed a different PQ model, called PX model [11] [12]. According to the proposal of these authors, the machine can be simulated by means of a PQ bus, with a given value of the active power P and where $Q = 0$. The reactive power consumed by the machine can be calculated by means of a reactance, for which they propose the value of the magnetizing reactance of the machine. This reactance, $\underline{Z} = jX_m$ is introduced as an admittance, $\underline{Y} = \frac{1}{jX_m}$ in the admittance matrix, and once the LF process is solved, allows the calculation of the reactive power as $Q = \frac{V^2}{X_m}$, where $\underline{V}$ is the voltage of the bus and $Q > 0$ if Q is considered as consumed reactive power.

The approximation can be good for values of power close to the nominal power, but the level of accuracy decreases if the power is far from that value. The advantage is that it allows a very quick calculation by means of a not very complicated model.

A small correction can be made in this model by considering $X = X_s + X_m$ as the value of the reactance X.

V.REACTIVE POWER DEPENDENT MODELS

In this section some models are presented where the reactive power is calculated as a function of some of the machine parameters and the voltage of the connection point to the system, which can be read in [7].

A first one is based on a simplification of the single-phase equivalent steady-state model of the induction machine, which is used for estimating the reactive power, as a function of the terminal voltage (i.e., stator voltage), and the active power. This simplification has been represented in Fig. 2, and in the configuration of such a simplification it can be assumed that $\underline{I}_s = \underline{I}_r$. All this allows Equations 2 and 3 to be written as expressions for the active and reactive powers. For obtaining these equations the theorem of complex power conservation has been taken into account.

$$P = P_m - I_r^2 R \tag{2}$$

$$Q = \frac{X_c - X_m}{X_m X_c} V^2 + I_r^2 X \tag{3}$$

where R and X are the resistance and reactance of the machine, both being the sum of the terms corresponding to stator and rotor, i.e., $R = R_s + R_r$ and $X = X_s + X_r$.

According to Fig. 2, the use of the theorem of complex power conservation leads to Equation 4.

$$\underline{V}\underline{I}_r^* = (R + jX)I_r^2 - P_m \tag{4}$$

By operating with the squares of the rms values, Equation 4 can also be written in the form of Equation 5.

$$\left(R^2 + X^2\right) I_r^4 - \left(V^2 + 2P_m R\right) I_r^2 + P_m^2 = 0 \tag{5}$$

Equation 5 can be used for obtaining the value of I_r^2 and, consequently, for obtaining Equation 6, by introducing this value in Equation 3.

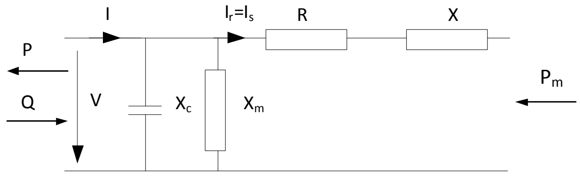

Fig. 2 An approximation of the steady-state model of the induction machine

$$Q = \frac{X_c - X_m}{X_c X_m} V^2 + \left(\frac{V^2 + 2RP_m}{2(R^2 + X^2)} - \frac{\sqrt{(V^2 + 2RP_m)^2 - 4P_m^2(R^2 + X^2)}}{2(R^2 + X^2)} \right) X \tag{6}$$

Empirically, the reactive power of the WT can be obtained as a function of the active power. This has been proposed by researchers of the Risø institute [13], such as can be seen in Equation 7, which can be considered a very accurate way for calculating the reactive power.

$$Q = -Q_0 - Q_1 P - Q_2 P^2 \tag{7}$$

where Q_0, Q_1 and Q_2 are values that can be experimentally obtained, so they should be obtained for each WT.

An approximation of Equation 6 to 7 can be operated by means of the MacLaurin polynomial (coefficients in Appendix), by taking into account the first two derivatives of Equation 6, and by neglecting the resistance R. In this case, with $R = 0$ from Equation 2, the fact that $P = P_m$ can be assumed.

With all this, the expression for the reactive power can be written in Equation 8, which is another approximation to the calculation of the reactive power.

$$Q = \frac{X_c - X_m}{X_c X_m} V^2 + \frac{X}{V^2} P^2 \tag{8}$$

When using the PQ models that propose Equations 6 or 8, it must be understood that the reactive powers of these equations are the so-called specified reactive powers, Q^s. This means that the proposal consists of using those values, Equation 6, or in a more simplified way, Equation 8, as specified reactive power, and compare them with the calculated reactive power, Q^c, in order to check the degree of approximation and convergence.

VI.THE SIMPLIFIED MODEL

Another possible approach to Equations 6 and 7, can be made by adding a new assumption, i.e., a value of $V = 1$ for the calculation of the reactive power Q in the previous model, which gives Equation 9 as a result, with the only advantage that there is no need to update its value in each iteration, which means the initial value of Q given by Equation 9, now the specified reactive power, Q^s, is a constant value through the process.

$$Q = \frac{X_c - X_m}{X_c X_m} + XP^2 \tag{9}$$

As a summary of all the explained above, the proposal is to use PQ models based on the parameters of the machine, where the active power P is obtained from the wind with the help of the WT power curve, and the reactive power can be obtained with different degrees of approximation, following Equations 6, 8 or 9.

VII.THE PI-EQUIVALENT MODEL

This model was presented in [14] and consisted of performing a conversion of a T two-port circuit into an equivalent π two-port circuit. This can be made because the equivalent circuit of the machine, Fig. 1 can be seen as a T two-port circuit. The mentioned equivalence is well known and is represented in Fig. 3.

The equations that describe the conversion are $\underline{Z}_a = \underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_2 \underline{Z}_3}{\underline{Z}_1}$, $\underline{Z}_b = \underline{Z}_3 + \underline{Z}_1 + \frac{\underline{Z}_3 \underline{Z}_1}{\underline{Z}_2}$ and $\underline{Z}_c = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_3}$, where $\underline{Z}_1 = R_s + jX_s$, $\underline{Z}_2 = R_r + jX_r$ and $\underline{Z}_3 = jX_m$.

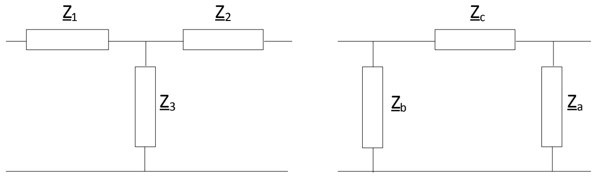

Fig. 3 T and π models of a two-port circuit

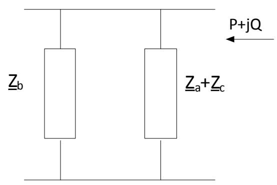

Fig. 4 Equivalent model of the asynchronous WT for the power flow analysis

After some operations the machine can be finally simulated as a PQ bus, following the model that can be seen in Fig. 4. The impedances $\underline{Z}_b$ and $\underline{Z}_a + \underline{Z}_c$ have to be introduced as admittances in the admittance matrix of the system.

The power exchanged between the machine and the network is given by Equation 10, where both active power P and reactive power Q are positive when injected into the network.

$$\underline{S} = P + jQ = \left(\frac{\underline{Z}_a}{\underline{Z}_a + \underline{Z}_c}\right)^* \left(\frac{\underline{Z}_a + \underline{Z}_c}{\underline{Z}_a} - \frac{\underline{Z}_c P_m}{V_r^2}\right) P_m \tag{10}$$

where, once again, P_m represents the mechanical power obtained from the wind.

Equation 10 has a real and an imaginary part. They are the active and reactive powers of a new version of a PQ model, in which they are considered the specified active and reactive powers.

It can be noticed that these values depend on the parameters of the machine, the mechanical power P_m, and the rotor voltage rms value, V_r, which is an unknown value up to here.

At this point the following consideration should be clarified. As mentioned in the introduction of the paper, the models presented are for short-circuited rotor machines. This concept makes one immediately think of a rotor voltage value $\underline{V}_r = 0$. However, in this model the rotor voltage denoted as $\underline{V}_r$ is the voltage at the resistance $R_r \frac{1-s}{s}$. This resistance can be used for expressing the value of the mechanical power as $P_m = -I_r^2 R_r \frac{1-s}{s}$, where the negative sign facilitates having positive power in generator operation.

Hence, the problem with this PQ bus model is its dependence on this rotor voltage, $\underline{V}_r$. To solve this inconvenience, the proposal is for this voltage to be obtained from the stator voltage $\underline{V}$ as can be seen in Equation 11.

$$\underline{V}_r = \underline{V} - \underline{Z}_c \left(-\left(\frac{\underline{S}_M}{\underline{V}}\right)^* - \frac{\underline{V}}{\underline{Z}_b}\right) \tag{11}$$

where $\underline{S}_M = \underline{S} - \underline{Y}_M^* V^2$, $\underline{Y}_M = \underline{Y}_b + \underline{Y}_{ac}$ and $\underline{Y}_{ac} = \frac{1}{\underline{Z}_a + \underline{Z}_c}$.

The model must be iteratively solved, in a similar way to other models commonly used in power system calculations.

VIII.THE MODEL BASED ON THE CONSERVATION OF COMPLEX POWER THEOREM

Coming back to the equivalent circuit of the asynchronous WT presented in Fig. 1, a different steady-stated model for LF analysis can be obtained, which in the authors' experience gives identical results with that given in section VII. This model has not been presented in previous papers, so it is presented in detail here.

It is based, one more time, on the conservation of complex power theorem and on obtaining Thevenin equivalent circuits of part of the network, and must be operated in an iterative way in the same manner as previous models.

For a given value of the terminal voltage, $\underline{V}$, the whole electrical network observed from the point of injection of mechanical power, P_m, can be substituted by its Thevenin equivalent circuit. This equivalent consists of a voltage source $\underline{E}_T$ behind an impedance $\underline{Z}_T$. The values of this source and impedance are given in Equations 12 and 13.

$$\underline{E}_T = \frac{jX_m}{R_s + j\left(X_s + X_m\right)} \underline{V} \tag{12}$$

$$\underline{Z}_T = R_T + jX_T = R_r + jX_r + \frac{1}{\frac{1}{jX_m} + \frac{1}{R_s + jX_s}} \tag{13}$$

The active and reactive power balances in the machine can be described for each iteration as in Equation 14.

$$\underline{E}_T \underline{I}_r^* = I_r^2 \underline{Z}_T - P_m \tag{14}$$

where $\underline{I}_r$ is the current at the point of injection of mechanical power into the Thevenin equivalent network, i.e., the rotor current. The rms value of this current, I_r, can be easily calculated from Equation 14 and is given by $I_r = \sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a}}$, where $a = Z_T^2$, $b = -\left(E_T^2 + 2P_m R_T\right)$ and $c = P_m^2$.

$\underline{I}_r$ in Equation 14 is the value of the current flowing through the rotor of the machine. According to this, the complex power transferred from the rotor to the rest of the machine, which consists of the stator and the magnetizing reactance, can be calculated following Equations 15 and 16, although this value will not be needed expressly.

$$P_{r \to sm} = P_m - I_r^2 R_r \tag{15}$$

$$Q_{r \to sm} = -I_r^2 X_r \tag{16}$$

On the other hand, at the mechanical power injection point, the difference between voltage $\underline{V}_r$ and current $\underline{I}$ angles is 0, as the injection of reactive power at that point is 0, allowing a rms value for V_r to be considered that is equal to the one given in Equation 17.

$$\underline{S}_r = P_m = V_r I_r^* \Rightarrow \underline{V}_r = \left(\frac{P_m}{\underline{I}_r}\right)^* = \frac{P_m}{I_r} = V_r \tag{17}$$

The reactive power consumed by the magnetizing reactance can be calculated in Equation 18.

$$Q_m = \frac{V_m^2}{X_m} \tag{18}$$

The value of $\underline{V}_m$ can be calculated with the help of Equation 19.

$$\begin{aligned} \underline{V}_m &= -\left(R_r + jX_r\right)\underline{I}_r = -R_r I_r + V_r - jX_r I_r \Rightarrow \\ V_m^2 &= \left(-R_r I_r + V_r\right)^2 + X_r^2 I_r^2 \Rightarrow V_m^2 = \left(-R_r I_r + \frac{P_m}{I_r}\right)^2 + X_r^2 I_r^2 \end{aligned} \tag{19}$$

The complex power injected into the stator circuit can be calculated in Equations 20 and 21.

$$P_{r \to s} = P_{r \to sm} = P_m - I^2 R_r \tag{20}$$

$$Q_{r \to s} = Q_{r \to sm} - Q_m = -\frac{V_m^2}{X_m} - I_r^2 X_r \tag{21}$$

Now, the calculation of the stator current $\underline{I}$ follows a process similar to the calculation of $\underline{I}_r$, i.e., $I = \sqrt{\frac{-b_1-\sqrt{b_1^2-4a_1c_1}}{2a_1}}$, where $a_1 = R_s^2 + X_s^2$, $b_1 = -\left(V^2 + 2P_{r\to s}R_s + 2Q_{r\to s}X_s\right)$ and $c_1 = P_{r\to s}^2 + Q_{r\to s}^2$, and finally the complex power injected into the network, i.e., what are conventionally called specified active and reactive powers of the PQ model are presented in Equations 22 and 23.

$$P = P_{r \to s} - R_s I_s^2 \tag{22}$$

$$Q = Q_{r \to s} - X_s I_s^2 + B_c V^2 \tag{23}$$

where $B_c = \frac{1}{X_c}$ is the value of the susceptance of the capacitor bank in parallel with the machine.

The model is a PQ model where specified active and reactive powers are given by Equations 22 and 23.

IX.IMPEDANCE-BASED MODEL

The other method proposed by Feijóo and Cidrás in [7] consists of modelling the machine as a $\underline{Z}$ bus, following the next three steps:

1. To calculate the power that each WT can extract from the wind for a given wind speed and a given rotor speed, according to its power coefficient curve.
2. To calculate the power that each WT can generate, according to the results of the load flow analysis, and to the rotor speed given in the previous step, Equation 1.
3. To both compare powers and look for the value of the slip, for which the electrical and the mechanical powers coincide, for the given wind speed.

First of all, it is appropriate to remark that the original paper [7] contained an error that was later corrected in an erratum in [15], information that can be important for those interested in reading that paper. The model was originally called RX although it is also referred to as $\underline{Z}$ model.

It is based on the steady-state model of the induction machine, represented by means of the impedance $\underline{Z} = R_s + jX_s + jX_m \frac{R_r + jsX_r}{R_r + js(X_r + X_m)}$, following Fig. 1, and a shunt capacitor with susceptance $B_c = \frac{1}{X_c}$, like in section VIII.

A previous and approximated way of working can consist of establishing a value of the mechanical power, P_m, and assuming that the slip can be calculated from Equation 24.

$$s = \frac{-V^2 R_r + \sqrt{V^4 R_r^2 - 4P_m R_r^2 \left(P_m X^2 + V^2 R_r\right)}}{2\left(P_m X_r + V^2 R_r\right)} \tag{24}$$

Equation 24 can be obtained from the steady-state model of the machine, Fig. 2, by applying the theorem of complex power conservation, and assuming the approximation that the stator resistance is 0. By assuming this value for the slip, the complete model of the machine can be used as an impedance in front of the network, so final values of the active power, P, and reactive power, Q, can be obtained at the end of the process, i.e., once the voltage has been obtained.

But a more exact way of operation can be carried out, by means of an iterative process, as follows. An initial value of the slip is considered at the beginning of the process, a suitable value of which can be the machine nominal slip, or alternatively the value obtained from Equation 24. With this value introduced in the expression for $\underline{Z}$, an initial load flow analysis can be carried out.

With the results obtained following this process, the mechanical power of the machine can be calculated as $P_m = -I_r^2 R_r \frac{1-s}{s}$, where I_r is the rms value of the rotor current, calculated from the rms value of the stator current I_s as $\underline{I}_r = \left(\frac{jsX_m}{R_r} + j\left(X_r + X_m\right)\right)\underline{I}$ where $\underline{I}_r$ and $\underline{I}_s$, are the phasors for the rotor and stator currents, following the notation established for this paper. Also, the sign of the mechanical power has been considered as positive when the machine behaves as a generator.

On the other hand, taking into account the wind speed and the slip of the machine, the value of the power coefficient can be calculated remembering the relationship $c_p = f(\lambda, \beta)$, for which the power extracted from the wind can be obtained from Equation 2.

When these two powers are not equal, then a new Newton-Raphson (NR) process looking for the convergence of both values begins. The slip is modified by means of Equation 25.

$$s_k = s_{k-1} + \Delta s \tag{25}$$

where s_{k-1} and s_k are the slip at the moment of the calculation and the new slip for the next iteration of the load flow analysis, respectively, and:

$$\Delta s = J^{-1} \Delta P_m \tag{26}$$

where ΔP_m is the difference between both powers, i.e., a term similar to the difference between specified and calculated powers generally used in the LF analysis.

If the approximation $\frac{1-s}{s} \approx \frac{1}{s}$ is assumed in the proximity of the operating point, J can be calculated as Equation 27.

$$J = \left(\frac{S}{V}\right)^2 \frac{R_r X_m^2 \left(R_r^2 - s^2\left(X_r + X_m\right)^2\right)}{\left(R_s^2 + s^2\left(X_r + X_m\right)^2\right)^2} \tag{27}$$

In Equation 27, $S = \sqrt{P_g^2 + Q_c^2}$, and P_g and Q_c are the active power generated and the reactive power consumed by the machine. These powers can be written as $P_g = -\frac{V^2}{Z^2}\Re\{\underline{Z}\}$ and $Q_g = -\frac{V^2}{Z^2}\Im\{\underline{Z}\}$, $\Re$ and $\Im$ meaning real and imaginary part.

X.COMPARISON OF MODELS

In this final section some comments about pros and cons of the models are presented. For the first six models the active power can be estimated or given as a function of the wind speed according to the WT data sheet given by the manufacturer. The seventh model is more complex.

Conventional PQ model given in section III. As its notation indicates, it is a conventional PQ model, i.e., there is no difference between it and a PQ model for a different load. The main advantage is its robustness, and the disadvantage is that it may be not very accurate, especially in the calculation of reactive power. It requires no additional iterations.

PX model given in section IV. This is a PQ model, but allows, in a way, a certain degree of approximation in reactive power calculation. However, it can be ensured only when the active power is close to the nominal one, which is not the typical case when simulating WTs.

Reactive power dependent model given in section V. It is another PQ model, but with the advantage that reactive power can be calculated with a good degree of approximation on the basis of the knowledge of the machine parameters and as a function of the bus voltage. Furthermore it does not require extra iterations, as these values can be updated at each iteration of the LF analysis.

Simplified model given in section VI. This corresponds to the previous model but it is not iterative. The reactive power is calculated as a function of the machine parameters and with the assumption the bus voltage is constant. So, this model can be considered between the PQ model and the reactive power dependent model.

π **model** given in section VII. This model aims to simulate the machine as an admittance in parallel with a current source dependent on the mechanical power. The model has to be iterated but the process can be solved independently of the LF iterative process or included in it. Even in the case of iterating the model independently of the LF process, it requires 2-3 iterations in order to reach a solution, so it is not especially heavy. The model gives as accurate results as the reactive dependent model at least.

Model based on the conservation of complex power given in section VIII. This model consists only of solving the previous one in a different way. The results given by both models coincide exactly.

Impedance based model given in section IX. This is the most complex model, especially indicated for conventional asynchronous WECs. It is based on the parameters of the machine and the main difference with all the previous models is that the active power is not estimated from the power curve given by the manufacturer, but is calculated as a function of the power coefficient curve. The model requires extra iterations (3-4 in the case of a good slip initialization) but these iterations can be included in the general LF process. The results are very exact, at least when it is assumed that the single phase equivalent model of the machine describes accurately the behavior of the machine. One of the advantages of this model is that it obtains exact internal variables of the machine for a subsequent dynamic simulation.

XI.CONCLUSIONS

In this paper a review has been presented, of the asynchronous WT models that have been mostly used by the authors for LF analysis calculation.

All the models have been checked by the authors in different works, with good results. The PQ model based on the complex power theorem given in section VIII. has not been previously published, but we have checked it with identical results to those given by the model presented in section VII.

No results have been given in this review as they can be found in the original papers.

Finally, very brief comments have been made about a few models presented by other authors, that we have not checked, but which also seem to be interesting.

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DERIVATION OF CONSTANTS

A special case of the Taylor series (for $P = 0$) is the MacLaurin approximation:

$$Q\left(P\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^nQ}{dP^n}P^n = Q|_{P=0} + \frac{1}{1!}Q'|_{P=0}P + \frac{1}{2!}Q''|_{P=0}P^2 + \epsilon(P) \tag{28}$$

In our case the term $\epsilon(P)$ is neglected, and the coefficients for the three first terms of Equation 28 are given in Equations 29 to 31.

$$Q\left(0\right) = \frac{X_c - X_m}{X_c X_m}V^2 = -Q_0 \tag{29}$$

$$Q'\left(0\right) = 0 = Q_1 \tag{30}$$

$$Q''\left(0\right) = \frac{2X}{V^2} = -Q_2 \tag{31}$$

A further approximation can be to consider a constant value for V. When operating in per unit values, i. e. values referred to a given base, a common way of operation in steady-state analysis methods employed in electrical engineering, the use of such a constant value consists of making $V = 1$, and then obataining Equations 32 to 34.

$$Q\left(0\right) = \frac{X_c - X_m}{X_c X_m} = -Q_0 \tag{32}$$

$$Q'(0) = 0 = Q_1 \tag{33}$$

$$Q''(0) = 2X = -Q_2 \tag{34}$$

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